

How Good are the Common Core math Standards?

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By common consent, the California Math Standards are among the top three state standards in the nation.

Why should we adopt **CCSS (Common Core State Standards in mathematics)**?

Because a set of standards is a very complex object. No matter how good it is, there is always **plenty of room** for improvement.

Overall, the CCSS is short of perfect (of course), but it is **much** better than what we have got.

I was asked by CCSSO for a quote, and here is what I said:

The Common Core mathematics standards succeed in being both mathematically coherent and grade level appropriate. Overall, they are the best standards that I have seen in the past twenty years. If we can design a professional development program of the same caliber to go with these standards, then our nation will be making a substantial first step towards educational excellence in mathematics.

Do I still stand by this statement? Yes and no.

Yes, there isn't a word that I want to change.

No, because I wish I could have written more. I should have added the following;

The choice of the right kind of assessment is also critically important for CCSS to work.

We need intensive professional development of our math teachers and a rigorous assessment system in order to benefit from the CCSS. No pain, no gain.

In this sense, we have to acquire some perspective on today's discussion:

To adopt or not to adopt, that is *not* the real issue. The real issue is whether California has the resolve and the wisdom to do good for its children, and commit to a long term process of incremental improvement.

Professional development **in mathematics** requires immense funding and **real expertise**. At the moment we seem to have neither. So we must be willing to work our way up from scratch.

State assessment **in mathematics** cannot afford to have too large a gray area between right and wrong. **State** assessment should test basic competence and must be as unambiguous as possible. It should be a “driver’s test”: take it, forget it, and go on to learn real mathematics.

For this reason, please do not go anywhere near so-called authentic assessment.

But let me answer this question first: **Is the CCSS worth all the trouble?**

Yes.

Do they match international expectations?

Definitely, with room to spare.

Here is one criterion commonly used to judge a set of standards: Pick a topic and ask if it is taught in grade N. For example, **is algebra taught in grade 8?**

To do this question justice, we must first ask: what is “algebra”?

If it is taken to mean “all the usual topics in Algebra I”, the answer is NO for the CCSS. **But it is also NO for the Japanese curriculum**, and Japan is one of the most revered nations in mathematics education. The Japanese curriculum spreads the Algebra I topics over grades 8–9, and the same can essentially be said about CCSS.

By the way, the Pythagorean Theorem is a grade 7 topic in California, but it is not taken up in Japan until grade 9.

Does this make California two grades better than Japan?

I let you decide.

Still with the question, **“is algebra taught in grade 8?”** What is meant by “taught”?

If it means putting a collection of topics in that grade without regard to whether students are prepared for them, then the answer to the first question is **YES** for California, Massachusetts, and Minnesota.

If it means carefully preparing the groundwork for algebra to optimize students’ chance of achieving it, then I am afraid the answer is **NO** for all three states.

CCSS does about half of the Algebra I topics in grade 8, but not *all* the topics because, in order to **properly** preparing students for algebra, it spends part of grade 8 on the preparation.

The twin pillars that support Algebra I are **rational numbers** (i.e., fractions and negative fractions) and **similar triangles** (congruence and similarity). See *National Mathematics Advisory Panel Report* (2008).

CCSS gives precise *mathematical* guidance on how to develop both topics in grades 5-**8**. The detailed description of the grade-by-grade progressions of both topics is unprecedented.

These two topics, when done properly, give ample opportunity to acclimatize students to *precision* and *abstraction*, two qualities so essential to algebra.

However, the curriculum of grades 5–7 in all the state standards allows the **replacement** of much of the precise reasoning and abstraction in rational numbers by hands-on activities, picture-drawings, analogies, and metaphors, and omits similarity.

Why similar triangles? A popular test item is: Given two points in the coordinate plane, find the equation of the line passing through these points.

This is a very simple problem if students know about similar triangles. But at present students learn algebra without benefit of similar triangles, so they learn about equations of straight lines **by rote**, and they make mistakes.

This gap in the logical development of the mathematics leading up to Algebra I was explicitly pointed out on page 181 of the [Mathematics for California Public Schools](#) (2006 version). See lines 7–16.

In the CCSS, a sizable part of the standards in **Grade 8** is devoted to the study of congruence and similarity, emphasizing precise definitions and precise statements of facts, but avoiding a full-blown course on geometric proofs by also emphasizing geometric intuition using hands-on activities and verification by experiments.

The sole purpose of this excursion into geometry is to make students feel comfortable *applying* the basic facts about similar triangles (such as the AA criterion for similarity) so that they can deal with the graph of a straight line.

Now look at the other pillar: rational numbers (fractions and negative fractions). [Fraction-phobia](#) has been in full swing in our country for decades. No corrective measure can afford to be just *business-as-usual*. Let me illustrate.

How to add $\frac{7}{8} + \frac{5}{6}$?

Take the Least Common Denominator of 8 and 6, which is 24. Note that $24 = 3 \times 8$ and $24 = 4 \times 6$. Therefore

$$\frac{7}{8} + \frac{5}{6} = \frac{3 \times 7 + 4 \times 5}{24} = \frac{41}{24}$$

Did that make any sense to you?

Adding is supposed to “combine things”. Did you see any “combining” in this procedural description of $\frac{7}{8} + \frac{5}{6}$?

How is a child who has mastered adding whole numbers supposed to learn this?

Here are the California standards on the addition of fractions:

Gr5 NS 2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals:

2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Gr6 NS 2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

There is nothing wrong with them, but it *is* business-as-usual.

Here is how CCSS approaches the addition of fractions:

Grade 3 Understand a fraction as a number on the number line; represent fractions on a number line diagram. Explain equivalence of fractions in special cases.

First represent $1/n$ as the point next to 0 when $[0,1]$ is divided into n equal parts, then m/n is the m -th division point to the right, and therefore, identifying $1/n$ with $[0,1/n]$, m/n with $[0,m/n]$, m/n is

“ m copies of $1/n$ ”,

i.e., m/n is joining m copies of $1/n$ together.

Grade 4 Explain why a fraction a/b is **equivalent** to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size, i.e., same point on the number line.

Define addition of fractions as joining parts referring to the same whole. Then for two fractions with **like denominators**,
 $m/n + k/n = (m+k)/n$.

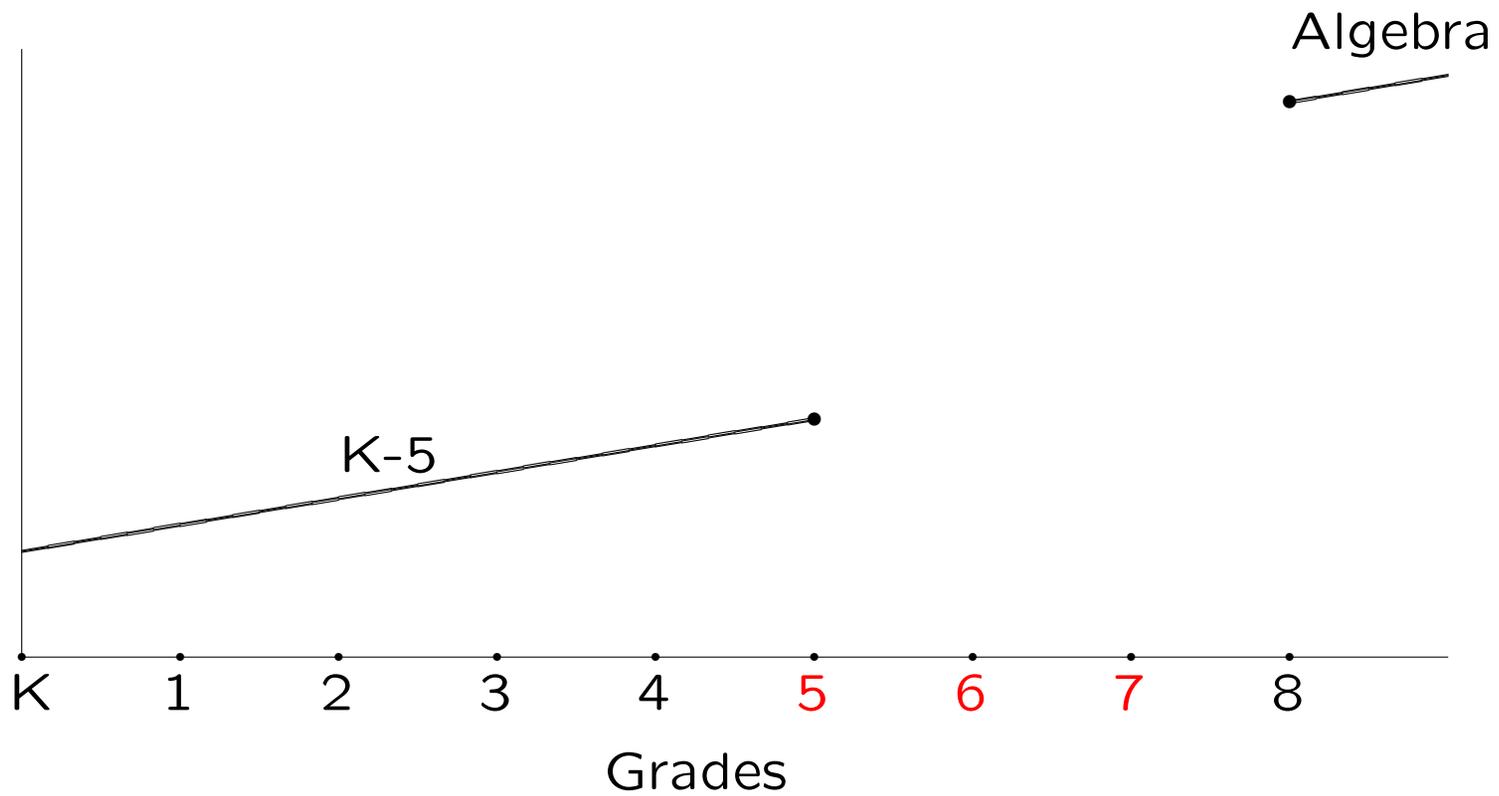
Grade 5 Add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions, so that we have fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

Altogether, these standards guide students through three grades to get them to know the meaning of adding fractions: Addition *is* putting things together, even for fractions, and the development ends with the formula $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$. No mention of LCD.

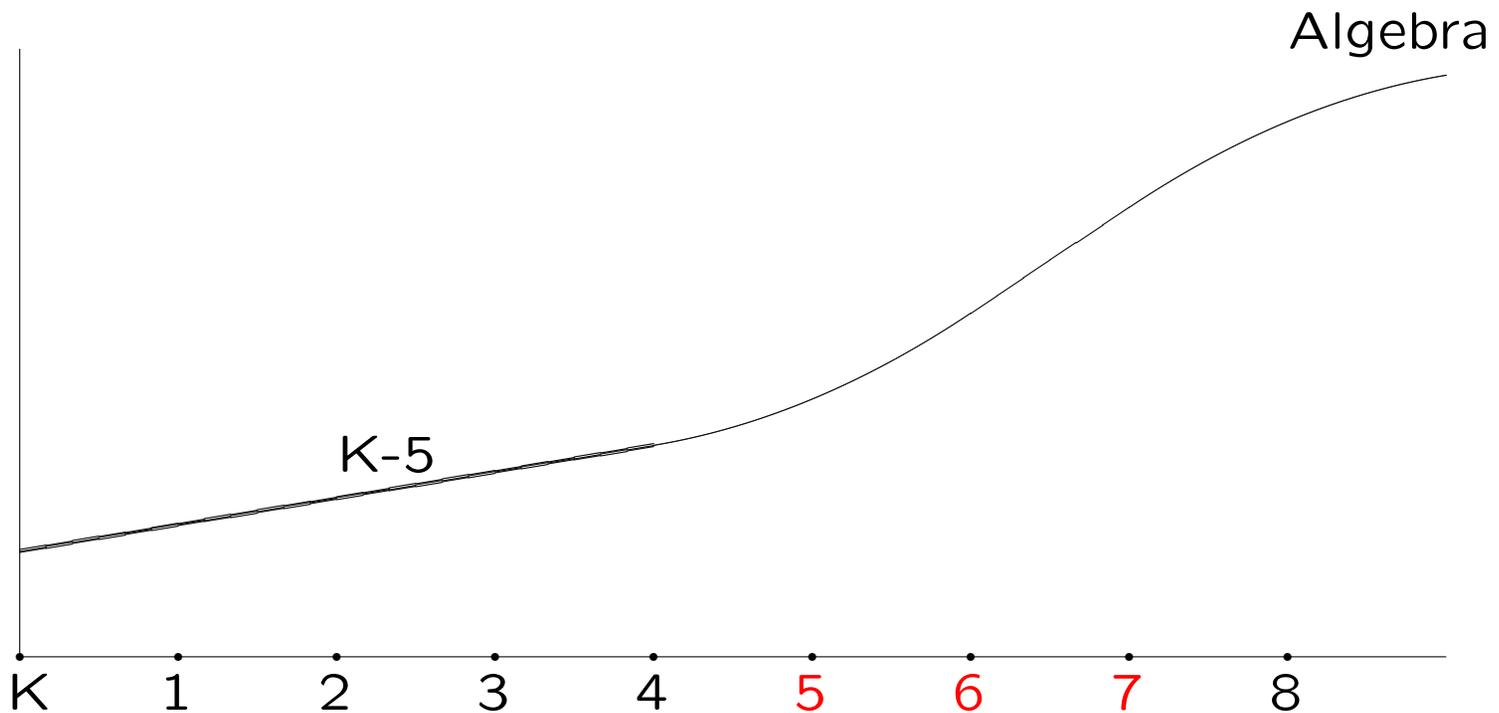
CCSS is full of delicate touches like this. It is hoped that CCSS will induce wholesale improvement in classroom instructions in California.

But we must not forget that only teachers who have the requisite content knowledge can make such improvement happen.

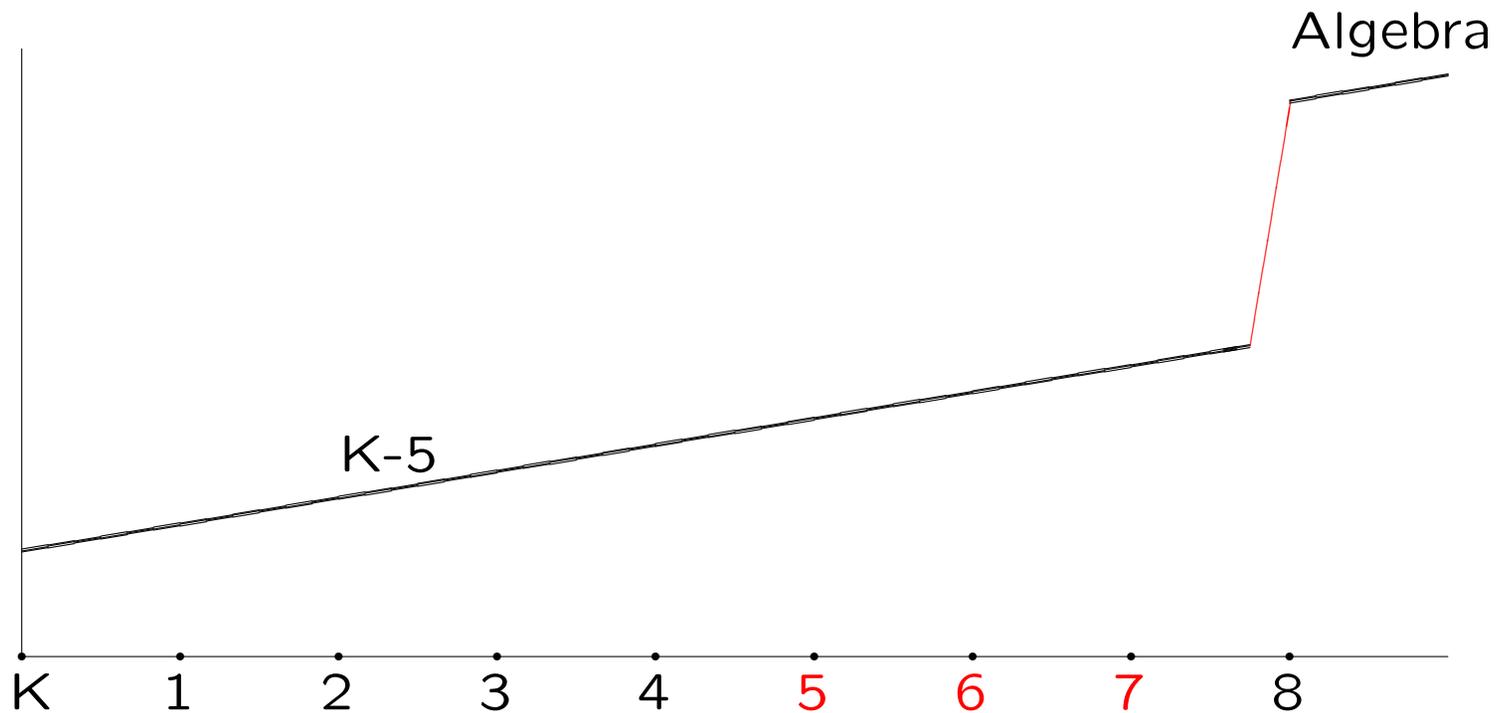
The K–4 school math curriculum is quite concrete, but algebra is on a higher level of abstraction. We depend on **Grades 5–7** to prepare students for algebra.



To go from grade 5 to grade 8, one **can** gradually elevate the level of sophistication to give students a smooth transition:



What the California curriculum does: Keep the level from grades 5 to 7 flat, and let students struggle in grade 8.



We are setting students up for failure.